

Making and Manipulating a Möbius Strip

A Möbius strip is a surprising object; it's a surface with only one edge and one side. There are many ways to make a Möbius strip: you can knit or crochet one, sew one from fabric, or construct one from paper and glue. It is worth trying different methods, as they each reveal something different about the Möbius strip. We'll use paper as it is the fastest way to make a Möbius strip, and unlike knitting, crochet, or fabric, it can easily be drawn on and cut!

Making a Möbius

Take a strip of rectangular paper – a 3-4cm strip cut lengthwise from a piece of A4 paper is ideal. Make a Möbius strip by bringing the two short edges together and giving one of them a *half-twist* (a twist of 180 degrees) before securely gluing or taping the two short edges together.



A paper Möbius strip.

Choose any point on the edge of the Möbius strip and place your finger on it. If you trace along the edge of the strip, you'll see that you trace around the entire edge before ending up back where you started. There's no top or bottom edge, it's all one edge! Now if you place your finger somewhere on the "outside" of the strip, and trace along the surface, you will soon see that your finger ends up on the "inside". You will arrive back at your starting point, but on the "inside" of the strip! If you keep going, your finger will eventually come back to where you started for a second time, and once again be on the "outside" of the strip. You have now journeyed over both the "inside" and the "outside" of the strip without ever taking your

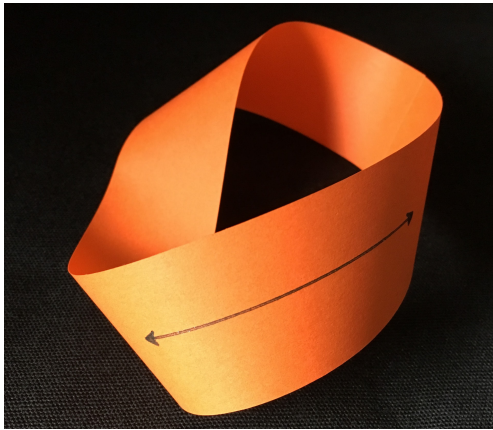
finger off the paper or crossing over the edge. This can only happen if the "inside" and the "outside" are not separate, forcing us to the curious conclusion that the Möbius strip has only one side.

We have just discovered that a Möbius strip has only one edge and one side. This is the perplexing property of the Möbius. Mathematicians call shapes with this property *non-orientable*. Can you think of any other non-orientable shapes?

Manipulating a Möbius

Some of the most unexpected properties of the Möbius strip are revealed only when we begin to manipulate it. Try the following experiments using a paper Möbius strip – you might want to use a fresh strip for each one. It's interesting to try to guess the outcome before attempting each experiment; our intuition is often a poor guide when it comes to the Möbius strip!

1. Take a pen or pencil and draw a line lengthwise along the centre of the Möbius strip (as shown below). Continue the line until it joins up with its starting point.



A line along the centre of the strip.



A line a third of the way down.

2. Take a pen or pencil and, starting about a third of the way down from the edge, draw a line lengthwise along the Möbius strip (as shown above). Continue the line until it joins up with its starting point.

What happens if you repeat the experiments above, and instead of merely drawing on the strip, you cut it? (You can reuse the strips from (1) and (2) if you need a line to guide you.)

3. Using scissors, cut lengthwise around the centre of the Möbius strip.
4. Using scissors, cut lengthwise around the Möbius strip, starting about a third of the way down from the edge.

What new shapes did you produce? Were they what you were expecting? Can you tape them back together to re-form the original Möbius strip? Do the new shapes have one side or two? What happens if you cut these new shapes as described in (3) or (4)?

Finally, we will construct a double Möbius.

5. Take two strips of paper and place one on top of the other. Holding both strips together, bring the two short edges together and give one of the ends a half-twist (just as you did when creating a single Möbius) before securely gluing or taping the two ends together (as shown below).



A double Möbius strip.

Take a slim object like a knitting needle or toothpick, insert it between the two bands and gently move it along the gap. Do you ever encounter a join? Try experiments (1)-(4) on the double Möbius and note any differences from the single Möbius. Finally, try to slide the two bands apart. What do you get? Does this remind you of any of the other experiments? If so, what's the connection?

There are many more things to try with Möbius strips. For instance, how short is the shortest Möbius strip you can make from paper? Can you make Möbius strips from other shapes? Keep on experimenting!