MATHS CRAFT new zealand

Crocheting a Hyperbolic Plane

The Curvature of Surfaces

Mathematicians refer to any flat surface as a **plane**. We are surrounded by many examples of planes: pieces of paper, table tops, whiteboards, computer screens, and the walls and floors of buildings. If you place your finger anywhere on the surface of a plane, you will see that it stretches away completely flat in every direction without ever distorting or curving. So we say that planes have **zero curvature**. Compare this to something spherical like an orange. If you place your finger at almost any point on the surface of an orange, you will see that the surface curves away from your finger in every direction. Because the curvature is the same in every direction, this is called **positive curvature**. Now consider a saddle (or a Pringles chip). If you place your finger in the middle of a saddle, the surface curves upwards and away from your finger towards the front and back of the saddle, but downwards and away from your finger towards the sides of the saddle. Because the curvature at this point is in two different directions (both upwards and downwards), we call this **negative curvature**.

The curvature of the surface of most objects is a combination of zero, positive, and negative curvature, which means that the curvature will vary depending on where you place your finger on the surface. However, there are examples of objects which have the same curvature at every point, and we say that these surfaces have **constant curvature**. Planes are flat everywhere, so have constant zero curvature, and spheres have the same positive curvature everywhere, and so have constant positive curvature. Surfaces which have constant negative curvature are called **hyperbolic planes**.

Crocheting Negative Curvature

You can crochet a **hyperbolic plane** like the one in the photo on the left, using just two simple crochet stitches. In fact, it's very similar to the humble ruffle used often in crochet and knitting! The instructions below are based on those developed by mathematics professor Daina Taimina. Daina is the author of the book "Crocheting Adventures with Hyperbolic Planes" which serves as both a gentle introduction to hyperbolic geometry and a creative crochet pattern book.

To crochet a hyperbolic plane you will need a crochet hook and some yarn. Any yarn will work, but for the best results, Daina recommends a yarn that will not stretch too much. She also suggests choosing a hook size smaller than the one recommended on the yarn label. The tighter and more even the crochet stitches are, the better the hyperbolic plane will hold its shape.

The hyperbolic plane is constructed using a combination of **chain stitches** and **double crochet stitches**. (See our *Useful Crochet Stitches* handout for detailed instructions.) The ruffle effect is created by 'increasing' (adding an extra stitch) at regular intervals. You can choose the interval, but somewhere between 3 and 12 stitches is a good guide. We'll call this number **N** in our instructions (in the photos below, **N** is 5). The smaller **N** is, the more increases you will do, and the more ruffled your plane will be.

1. Crochet a foundation chain of between 10 and 20 stitches (below, left). This is the first row.
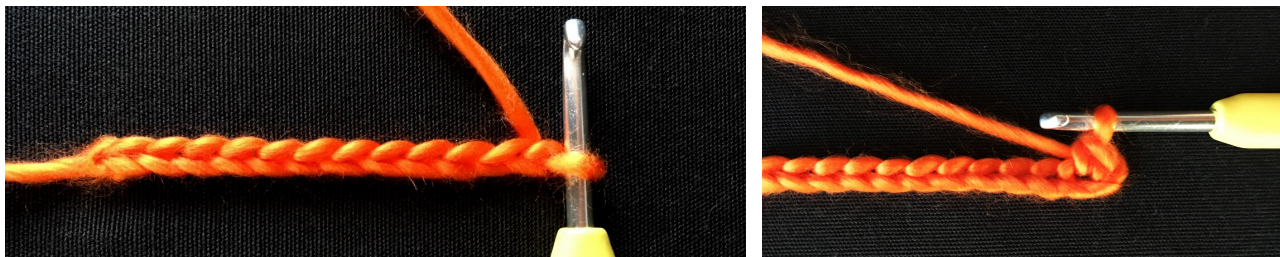
2. For the **first stitch** in the next row, insert the hook into the second chain from the hook and make a double crochet stitch (above, right). (See our *Useful Crochet Stitches* handout for instructions.)

3. For the **next N stitches** continue to double crochet, inserting the hook into the chain stitch directly beside your previous stitch each time (below, left).
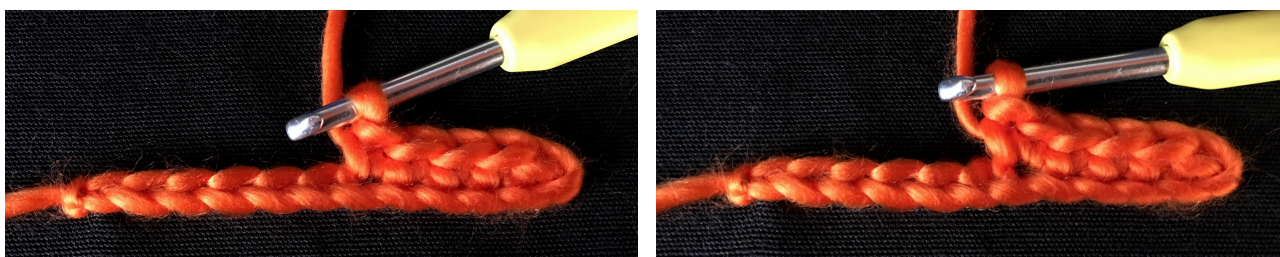
4. For the **next stitch** (stitch **N+1**) proceed as before except instead of inserting the hook into a new chain stitch, insert the hook into the same chain as the previous stitch (the **N**th stitch). There are now two double crochet stitches in the same chain (above, right). This is a crochet increase.

5. **Repeat Steps 3 and 4** until you reach the end of the row. At the end of the row make one chain stitch (below, left). Turn your work over - the chain stitch should now be at the start of the next row. You will notice that your work is starting to curve.
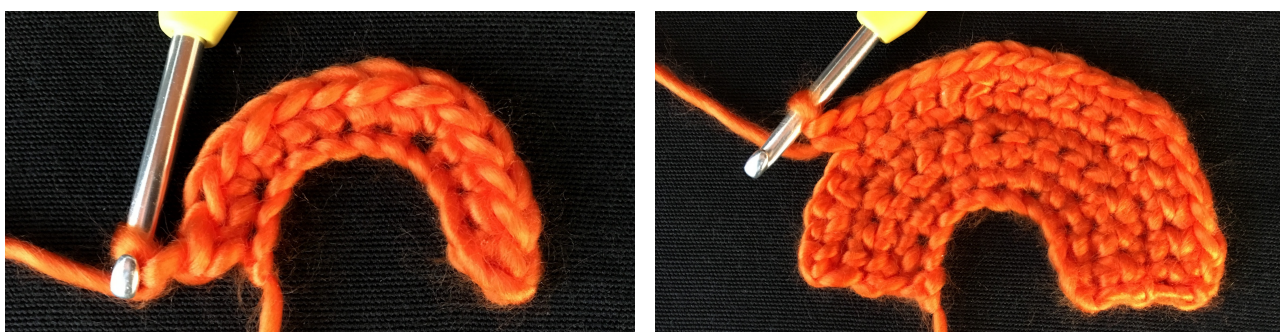
6. **Repeat Steps 3-5** until you have made a hyperbolic plane of the size you desire. Note that instead of crocheting into the foundation chain, from now on you will be crocheting into the previous row of double crochet stitches (above, right). As your plane grows, it will stop lying flat and start to ruffle.

7. To finish, complete the current row and cut the working yarn, leaving a long tail. Remove your hook from the loop and pull the working yarn tail through the loop.

For more hyperbolic crochet, visit Daina's website at: www.math.cornell.edu/~dtaimina/.